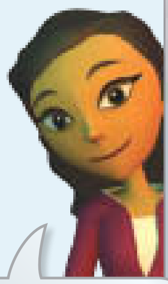
4-6 Congruence in Right Triangles

Common Core State Standards

G-SRT.B.5 Use congruence . . . criteria to solve problems and prove relationships in geometric figures.

MP 1, MP 3

Objective To prove right triangles congruent using the Hypotenuse-Leg Theorem

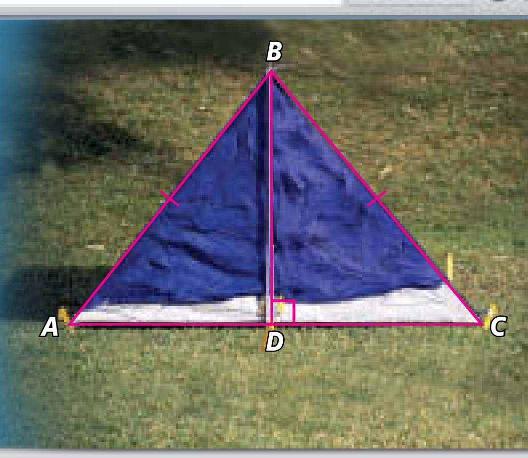


What does the large triangle tell you about angles in the figure?

MATHEMATICAL PRACTICES

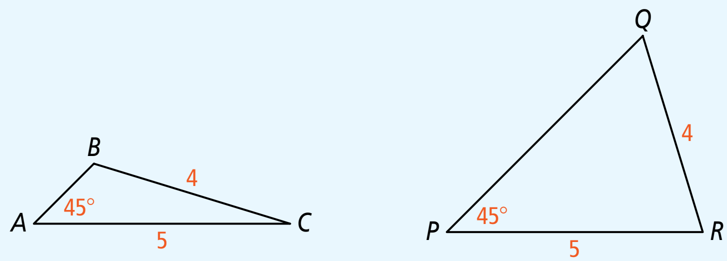
SOLVE IT! Getting Ready!

One of the tent flaps was damaged in a storm. Can you use the other flap as a pattern to replace it? Explain.



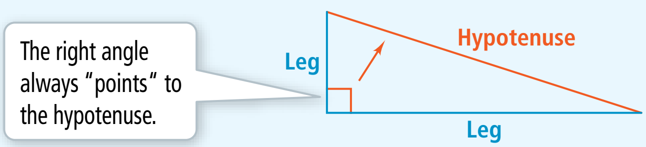
Lesson Vocabulary
- hypotenuse
- legs of a right triangle

In the diagram below, two sides and a nonincluded angle of one triangle are congruent to two sides and the nonincluded angle of another triangle.



Notice that the triangles are not congruent. So, you can conclude that Side-Side-Angle is *not* a valid method for proving two triangles congruent. This method, however, works in the special case of right triangles, where the right angles are the nonincluded angles.

In a right triangle, the side opposite the right angle is called the **hypotenuse.** It is the longest side in the triangle. The other two sides are called **legs.**



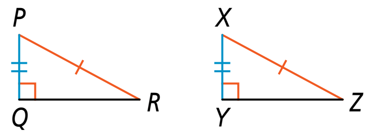
Essential Understanding You can prove that two triangles are congruent without having to show that *all* corresponding parts are congruent. In this lesson, you will prove right triangles congruent by using one pair of right angles, a pair of hypotenuses, and a pair of legs.

take note

Theorem 4-6 Hypotenuse-Leg (HL) Theorem

Theorem
If the hypotenuse and a leg of one right triangle are congruent to the hypotenuse and a leg of another right triangle, then the triangles are congruent.

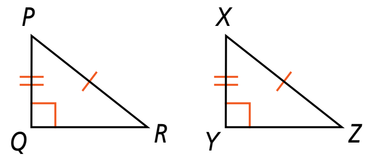
If . . .
$\triangle PQR$ and $\triangle XYZ$ are right $\triangle$s,
$\overline{PR} \cong \overline{XZ}$, and $\overline{PQ} \cong \overline{XY}$



Then . . .
$\triangle PQR \cong \triangle XYZ$

To prove the HL Theorem you will need to draw auxiliary lines to make a third triangle.

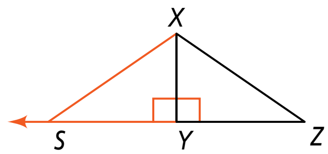
Proof Proof of Theorem 4-6: Hypotenuse-Leg Theorem



Given: $\triangle PQR$ and $\triangle XYZ$ are right triangles, with right angles Q and Y. $\overline{PR} \cong \overline{XZ}$ and $\overline{PQ} \cong \overline{XY}$.

Prove: $\triangle PQR \cong \triangle XYZ$

Proof: On $\triangle XYZ$, draw $\overrightarrow{ZY}$.



Mark point S so that $YS = QR$. Then, $\triangle PQR \cong \triangle XYS$ by SAS.

Since corresponding parts of congruent triangles are congruent, $\overline{PR} \cong \overline{XS}$. It is given that $\overline{PR} \cong \overline{XZ}$, so $\overline{XS} \cong \overline{XZ}$ by the Transitive Property of Congruence. By the Isosceles Triangle Theorem, $\angle S \cong \angle Z$, so $\triangle XYS \cong \triangle XYZ$ by AAS. Therefore, $\triangle PQR \cong \triangle XYZ$ by the Transitive Property of Congruence.

take note

Key Concept Conditions for HL Theorem

To use the HL Theorem, the triangles must meet three conditions.

Conditions

- There are two right triangles.
- The triangles have congruent hypotenuses.
- There is one pair of congruent legs.

Proof

Problem 1 Using the HL Theorem

On the basketball backboard brackets shown below, $\angle ADC$ and $\angle BDC$ are right angles and $\overline{AC} \cong \overline{BC}$. Are $\triangle ADC$ and $\triangle BDC$ congruent? Explain.

Plan

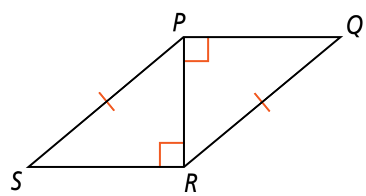
How can you visualize the two right triangles? Imagine cutting $\triangle ABC$ along $\overline{DC}$. On either side of the cut, you get triangles with the same leg $\overline{DC}$.

- You are given that $\angle ADC$ and $\angle BDC$ are right angles. So, $\triangle ADC$ and $\triangle BDC$ are right triangles.
- The hypotenuses of the two right triangles are $\overline{AC}$ and $\overline{BC}$. You are given that $\overline{AC} \cong \overline{BC}$.
- $\overline{DC}$ is a common leg of both $\triangle ADC$ and $\triangle BDC$. $\overline{DC} \cong \overline{DC}$ by the Reflexive Property of Congruence.

Yes, $\triangle ADC \cong \triangle BDC$ by the HL Theorem.

Got It? **1. a. Given:** $\angle PRS$ and $\angle RPQ$ are right angles, $\overline{SP} \cong \overline{QR}$

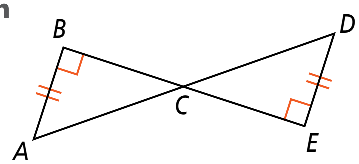
Prove: $\triangle PRS \cong \triangle RPQ$



b. Reasoning Your friend says, "Suppose you have two right triangles with congruent hypotenuses and one pair of congruent legs. It does not matter which leg in the first triangle is congruent to which leg in the second triangle. The triangles will be congruent." Is your friend correct? Explain.

Proof

Problem 2 Writing a Proof Using the HL Theorem



Given: $\overline{BE}$ bisects $\overline{AD}$ at C,
$\overline{AB} \perp \overline{BC}$, $\overline{DE} \perp \overline{EC}$, $\overline{AB} \cong \overline{DE}$

Prove: $\triangle ABC \cong \triangle DEC$

Plan

How can you get started?
Identify the hypotenuse of each right triangle. Prove that the hypotenuses are congruent.

$\overline{BE}$ bisects $\overline{AD}$. → $\overline{AC} \cong \overline{DC}$
Given → Def. of bisector

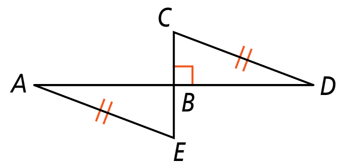
$\overline{AB} \perp \overline{BC}$, $\overline{DE} \perp \overline{EC}$ → $\angle ABC$ and $\angle DEC$ are right $\angle$s. → $\triangle ABC$ and $\triangle DEC$ are right $\triangle$s.
Given → Def. of $\perp$ lines → Def. of right triangle

$\overline{AB} \cong \overline{DE}$
Given

$\overline{AC} \cong \overline{DC}$, right triangles, $\overline{AB} \cong \overline{DE}$ → $\triangle ABC \cong \triangle DEC$
HL Theorem

Got It? **2. Given:** $\overline{CD} \cong \overline{EA}$, $\overline{AD}$ is the perpendicular bisector of $\overline{CE}$

Prove: $\triangle CBD \cong \triangle EBA$

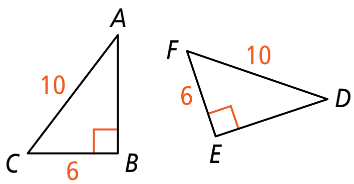


Lesson Check

Do you know HOW?

Are the two triangles congruent? If so, write the congruence statement.

1.



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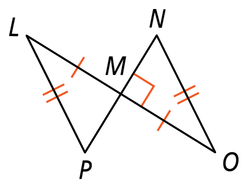

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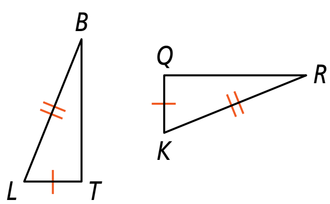

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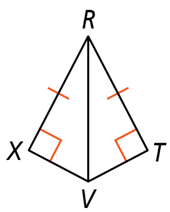
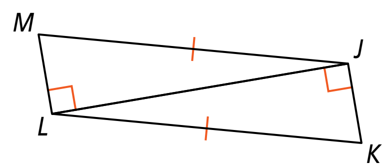


Do you UNDERSTAND? MATHEMATICAL PRACTICES

5. Vocabulary A right triangle has side lengths of 5 cm, 12 cm, and 13 cm. What is the length of the hypotenuse? How do you know?

6. Compare and Contrast How do the HL Theorem and the SAS Postulate compare? How are they different? Explain.

7. Error Analysis Your classmate says that there is not enough information to determine whether the two triangles below are congruent. Is your classmate correct? Explain.



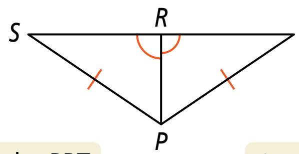
Practice and Problem-Solving Exercises

MATHEMATICAL PRACTICES

A Practice

8. **Developing Proof** Complete the flow proof.

See Problem 1.



Given: $\overline{PS} \cong \overline{PT}$, $\angle PRS \cong \angle PRT$

Prove: $\triangle PRS \cong \triangle PRT$

$\angle PRS$ and $\angle PRT$ are $\cong$.
Given

$\angle PRS$ and $\angle PRT$ are supplementary.
$\measuredangle$s that form a linear pair are supplementary.

$\angle PRS$ and $\angle PRT$ are right $\measuredangle$s.
a. ?

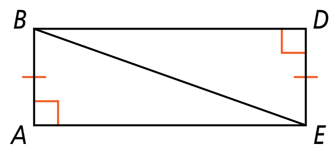
$\triangle PRS$ and $\triangle PRT$ are right $\triangle$s.
b. ?

$\overline{PS} \cong \overline{PT}$
c. ?

$\overline{PR} \cong \overline{PR}$
d. ?

$\triangle PRS \cong \triangle PRT$
e. ?

9. **Developing Proof** Complete the paragraph proof.



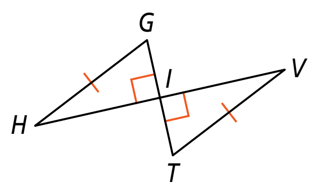
Given: $\angle A$ and $\angle D$ are right angles, $\overline{AB} \cong \overline{DE}$

Prove: $\triangle ABE \cong \triangle DEB$

Proof: It is given that $\angle A$ and $\angle D$ are right angles. So, **a.** ? by the definition of right triangles. **b.** ? , because of the Reflexive Property of Congruence. It is also given that **c.** ? . So, $\triangle ABE \cong \triangle DEB$ by **d.** ? .

10. Proof **Given:** $\overline{HV} \perp \overline{GT}$, $\overline{GH} \cong \overline{TV}$,
I is the midpoint of $\overline{HV}$

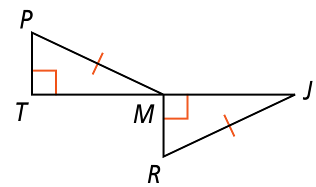
Prove: $\triangle IGH \cong \triangle ITV$



11. Proof **Given:** $\overline{PM} \cong \overline{RJ}$,
$\overline{PT} \perp \overline{TJ}$, $\overline{RM} \perp \overline{TJ}$,
M is the midpoint of $\overline{TJ}$

Prove: $\triangle PTM \cong \triangle RMJ$

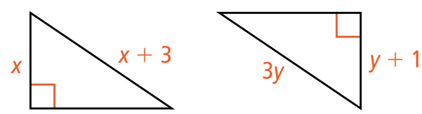
See Problem 2.



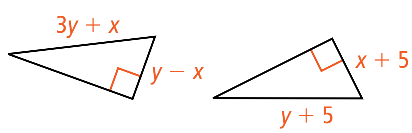
B Apply

Algebra **For what values of x and y are the triangles congruent by HL?**

12.



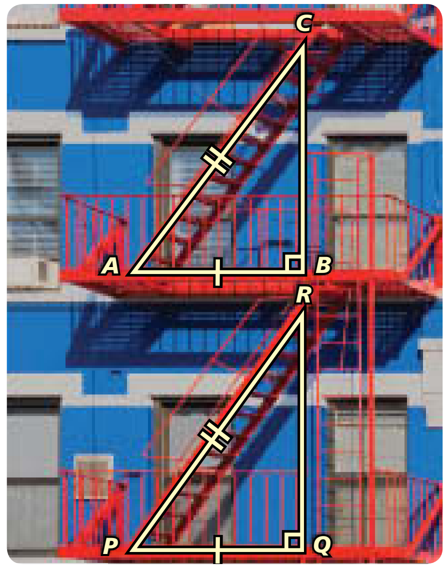
13.



14. Study Exercise 8. Can you prove that $\triangle PRS \cong \triangle PRT$ without using the HL Theorem? Explain.

15. Think About a Plan $\triangle ABC$ and $\triangle PQR$ are right triangular sections of a fire escape, as shown. Is each story of the building the same height? Explain.

- What can you tell from the diagram?
- How can you use congruent triangles here?

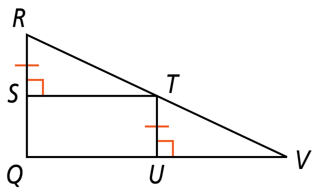


16. Writing "A HA!" exclaims your classmate. "There must be an HA Theorem, sort of like the HL Theorem!" Is your classmate correct? Explain.

17. Given: $\overline{RS} \cong \overline{TU}$, $\overline{RS} \perp \overline{ST}$, $\overline{TU} \perp \overline{UV}$, T is the midpoint of $\overline{RV}$

Proof

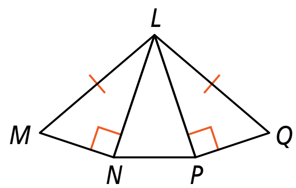
Prove: $\triangle RST \cong \triangle TUV$



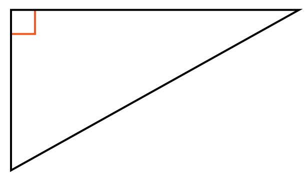
18. Given: $\triangle LNP$ is isosceles with base $\overline{NP}$, $\overline{MN} \perp \overline{NL}$, $\overline{QP} \perp \overline{PL}$, $\overline{ML} \cong \overline{QL}$

Proof

Prove: $\triangle MNL \cong \triangle QPL$



Constructions **Copy the triangle and construct a triangle congruent to it using the given method.**



19. SAS

20. HL

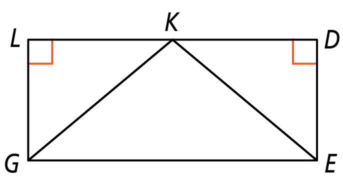
21. ASA

22. SSS

23. Given: $\triangle GKE$ is isosceles with base $\overline{GE}$, $\angle L$ and $\angle D$ are right angles, and K is the midpoint of $\overline{LD}$.

Proof

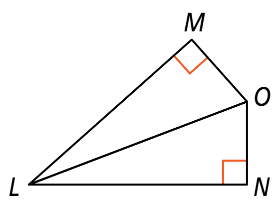
Prove: $\overline{LG} \cong \overline{DE}$



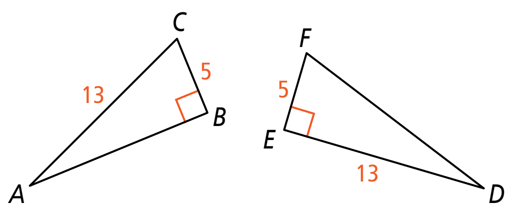
24. Given: $\overline{LO}$ bisects $\angle MLN$, $\overline{OM} \perp \overline{LM}$, $\overline{ON} \perp \overline{LN}$

Proof

Prove: $\triangle LMO \cong \triangle LNO$



25. Reasoning Are the triangles congruent? Explain.



26. a. **Coordinate Geometry** Graph the points $A(-5, 6)$, $B(1, 3)$, $D(-8, 0)$, and $E(-2, -3)$. Draw $\overline{AB}$, $\overline{AE}$, $\overline{BD}$, and $\overline{DE}$. Label point C, the intersection of $\overline{AE}$ and $\overline{BD}$.

b. Find the slopes of $\overline{AE}$ and $\overline{BD}$. How would you describe $\angle ACB$ and $\angle ECD$?

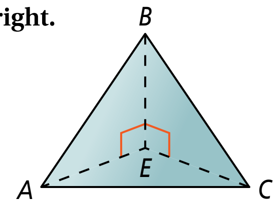
c. **Algebra** Write equations for $\overleftrightarrow{AE}$ and $\overleftrightarrow{BD}$. What are the coordinates of C?

d. Use the Distance Formula to find AB, BC, DC, and DE.

e. Write a paragraph to prove that $\triangle ABC \cong \triangle EDC$.

Challenge

Geometry in 3 Dimensions **For Exercises 27 and 28, use the figure at the right.**



27. **Given:** $\overline{BE} \perp \overline{EA}$, $\overline{BE} \perp \overline{EC}$, $\triangle ABC$ is equilateral

Proof

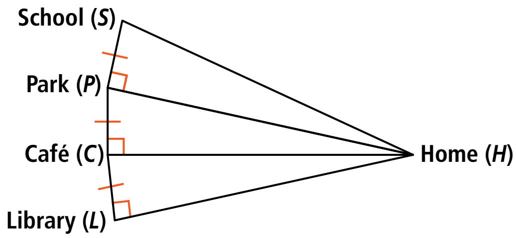
Prove: $\triangle AEB \cong \triangle CEB$

28. **Given:** $\triangle AEB \cong \triangle CEB$, $\overline{BE} \perp \overline{EA}$, $\overline{BE} \perp \overline{EC}$

Can you prove that $\triangle ABC$ is equilateral? Explain.

Standardized Test Prep

SAT/ACT



29. You often walk your dog around the neighborhood. Based on the diagram at the right, which of the following statements about distances is true?

(A) $SH = LH$ (C) $SH > LH$

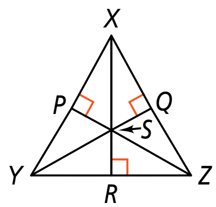
(B) $PH = CH$ (D) $PH < CH$

30. What is the midpoint of $\overline{LM}$ with endpoints $L(2, 7)$ and $M(5, -1)$?

(F) (3.5, 3) (H) (2, 4.5)

(G) (3.5, 4) (I) (7, 6)

Short Response



31. In equilateral $\triangle XYZ$, name four pairs of congruent right triangles. Explain why they are congruent.

Mixed Review

For Exercises 32 and 33, what type of triangle must $\triangle STU$ be? Explain. See Lesson 4-5.

32. $\triangle STU \cong \triangle UTS$

33. $\triangle STU \cong \triangle UST$

Get Ready! **To prepare for Lesson 4-7, do Exercises 34–36.**

Can you conclude that the triangles are congruent? Explain. See Lessons 4-3 and 4-6.

34. $\triangle ABC$ and $\triangle LMN$

35. $\triangle LMN$ and $\triangle HJK$

36. $\triangle RST$ and $\triangle ABC$